Wednesday, February 15

Written Test 1 Review

Given two sets S and T, say we write:

- S $\dot{\vee}$ T for their union
- S $\dot{\wedge}$ T for their intersection
- S \ T for their difference

pow.

$\mathbb{P}$.

What is the **cardinality** of the power set of ({a, b, c, d} \ {a, e}) $\vee$ {a, f}? Enter an integer value (with no spaces).

$$\binom{n}{m} = \frac{n!}{(n-m)!\,m!}$$

$$\left|\mathbb{P}\left(\underbrace{(\{a,b,c,d\} \setminus \{a,e\})}_{\{b,c,d\}} \vee \{a,f\}\right)\right|$$

$$\{b,c,d\} \vee \{a,f\}$$

$$\mathbb{P}(\{a,b,c,d,f\})$$

$$2^5 = 32$$

$$\binom{5}{2} = \frac{5 * 4}{2!}$$

How many subsets in $\mathbb{P}$ are of card 2?

$$\binom{5}{2} = \frac{5 * 4}{2!} = 10$$

$$\binom{5}{3} = \binom{5}{2}$$

$$\boxed{\binom{n}{m} = \binom{n}{n-m}}$$

$\mathbb{P}(\{a, b, c, d, f\})$

$\{ s \mid s \in \mathbb{P}(\{a, b, c, d, f\}) \wedge |s| = 2 \}$

$\boxed{10}$

$\{a, b\}$ $\{b, c\}$ $\{c, d\}$ $\{d, f\}$

$\{a, c\}$ $\{b, d\}$ $\{c, f\}$

$\{a, d\}$ $\{b, f\}$

$\{a, f\}$

Consider the following logical quantification:

!x,y.x:NAT&y:NAT=>$\boxed{\text{x+y>=10\&x+y<20}}$

Convert the above predicate to an equivalent one using the other logical quantifier.

Note the following constraints on your answer:

- Only put pairs of parentheses **when necessary**.
- Like the above predicate, there should be **no** white spaces.
- Like the above predicate, numerical constants (i.e., 10, 20) must appear as the right operands of the relational expressions (e.g., x + y >= 10).
- Relational expressions should be simplified whenever possible, e.g., write x >= 20 rather than not(x < 20).

Be cautious about the spellings: this question will be graded **automatically** and no partial marks will be give to spelling mistakes.

Answer:

The correct answer is: not#x,y.x:NAT&y:NAT&(x+y<10orx+y>=20))

$$\forall x \cdot R(x) \Rightarrow P(x)$$

$$\equiv \neg(\exists x \cdot R(x) \wedge \neg P(x))$$

de morgan:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(x+y \geq 10 \wedge x+y < 20)$$

$$\equiv \neg(x+y \geq 10) \vee \neg(x+y < 20)$$

$$\equiv x+y < 10 \vee x+y \geq 20$$

$$\{a, b, c, d\} \lhd \{(\underline{\underline{a}}, 2), (\underline{\underline{b}}, 3)\} = \{(a, 2), (b, 3)\}$$

$$S \lhd R = \{(x, y) \mid \boxed{(x, y) \in R} \land x \in S\}$$

only consider what's in R

((

Consider two sets:

- S = {~~x~~, y}
- T = {1, 2, ~~3~~}

Enumerate the following set:

{(a,b) | a : S & b : T & a /= x & b < 3}

Requirements. In your answer:

- Pairs must be **sorted** in an **ascending** order by the first elements, or by the second elements if the first elements are identical. For examples: (x, 2) appears before (y, 1), (x, 1) appears before (x, 2), etc.
- No white spaces should be included, e.g., write (x,1) rather than (x, 1).

Be cautious about the spellings: this question will be graded **automatically** and so no partial marks will be given due to spelling mistakes.

Answer: {(y,1),(y,2)} ✖

The correct answer is: {(y,1),(y,2)}

Consider two sets:

- S = {x, y}
- T = {1, 2, 3}

Consider r such that r : S <-> T:

{(x, 1), (x, 3), (y, 1), (y, 2)}

$\{(x,1),(x,3),(y,1)\}$

What is the result of the following expression:

$\{1,3\}$

{x} <<| (r |> (T \ {2})) $\triangleright$ $\{x\} \lhd \boxed{(r \triangleright (T \setminus \{2\}))}$

Requirements. In your answer:

- Pairs must be **sorted** in an **ascending** order by the first elements, or by the second elements if the first elements are identical. For examples: (x, 2) appears before (y, 1), (x, 1) appears before (x, 2), etc.
- No white spaces should be included, e.g., write (x,1) rather than (x, 1).

Be cautious about the spellings: this question will be graded **automatically** and so no partial marks will be given due to spelling mistakes.

$\{(y,1)\}$

Answer:

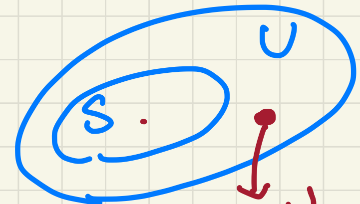
The correct answer is: {(y,1)}

$S = \{1, 2, 3\}$

$T = \{1, 3\}$

$U = \{1, 2, 3\}$

$S \subset U$



$\in U \notin S$

✓ **Subset**

$S \overset{\times}{\subseteq} T$ (3) $T \overset{✓}{\subseteq} S$ $S \overset{✓}{\subseteq} U$ $U \overset{✓}{\subseteq} S$

Proper subset

$S \overset{\times}{\subset} T$ $T \overset{✓}{\subset} S$ $S \overset{\times}{\subset} U$ $U \overset{\times}{\subset} S$

$$S \subset T \iff S \subseteq T \wedge |S| \overset{\times}{<} |T|$$

$\{a, b\}$ $\{1, 2, 3\}$

$r \in S \leftrightarrow T$

r satisfies functional property

↳ r is a partial function

↳ only those partial functions whose domain is S are total

$\{(a, 1), (b, 1)\}$

↳ total, not injective.